# POLY234

Display

(HP-41CX, Hewlett Packard 1983 and DM41X, SwissMicros 2020)
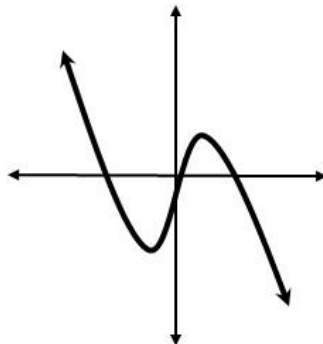
Overview[1]

Program `POLY234` program calculates (!) the roots of 2nd, 3rd and 4th degree polynomials. Both real and complex roots can be calculated on the following basis:
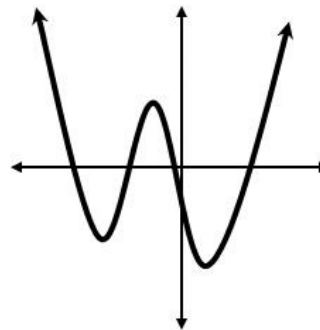
Degree	Polynomial expression	Method of calculation
2nd	$f(\mathbf{x}) = \mathbf{x}^2 + a_1\mathbf{x} + a_0$	Quadratic formula
3rd	$f(\mathbf{x}) = \mathbf{x}^3 + a_2\mathbf{x}^2 + a_1\mathbf{x} + a_0$	Cardano's algorithm
4th	$f(\mathbf{x}) = \mathbf{x}^4 + a_3\mathbf{x}^3 + a_2\mathbf{x}^2 + a_1\mathbf{x} + a_0$	Ferrari's algorithm
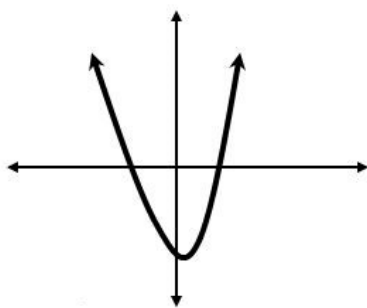
Quadratic Function
(deg. = 2)

Cubic Function
(deg. = 3)

Quartic Function
(deg. = 4)

Another method of finding the roots of polynomials is through the Newton–Raphson method which is not part of this program. The reason being that finding roots by convergence is not deterministic and requires a different approach to programming.

Please note my default FIX 5 and European (flag 28) setting in below examples.

Examples: 2nd degree polynomials

Find the (real) roots of the following polynomial: $f(\mathbf{x}) = \mathbf{x}^2 + 2\mathbf{x} - 15$

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[1] *This program is copyright and is supplied without representation or warranty of any kind. The author assumes no responsibility and shall have no liability, consequential or otherwise, of any kind arising from the use of this program material or any part thereof*

KEYSTROKES	DISPLAY	COMMENTS
		Run POLY234 for 2nd degree
[XEQ] [ALPHA]POLY234[ALPHA]	$DEGREE=?$	Enter degree of the polynomial
2[R/S]	$a0,a1=?$	Enter coefficients a_0=-15 and a_1=2
-15[ENTER]2[R/S]	$X1=3,00$	Root 1 (in Y register)
[R/S]	$X2=-5,00$	Root 2 (in Y register)

Find the (complex, based on i^2=-1) roots of the following polynomial: $f(x) = x^2 + 2x + 15$

KEYSTROKES	DISPLAY	COMMENTS
[R/S]	$0,00000$	Run POLY234 again
[R/S]	$DEGREE=?$	Enter degree of the polynomial
2[R/S]	$a0,a1=?$	Enter coefficients a_0=15 and a_1=2
15[ENTER]2[R/S]	$X1=-1,00+3,74I$	Root 1 (in X and Y register)
[R/S]	$X2=-1,00-3,74I$	Root 2 (in X and Y register)

Find the (complex) roots of the following polynomial: $f(x) = x^2 + 144$

KEYSTROKES	DISPLAY	COMMENTS
[R/S]	$-3,74166$	Run POLY234 again
[R/S]	$DEGREE=?$	Enter degree of the polynomial
2[R/S]	$a0,a1=?$	Enter coefficients a_0=144 and a_1=0
144[ENTER]0[R/S]	$X1=12,00I$	Root 1 (in X register)
[R/S]	$X2=-12,00I$	Root 2 (in X register)

Examples: 3rd degree polynomials

Find the (real) roots of the following polynomial: $f(x) = x^3 - 37x^2 - 205x + 12121$

KEYSTROKES	DISPLAY	COMMENTS
		Run POLY234 for 3rd degree
[XEQ] [ALPHA]POLY234[ALPHA]	$DEGREE=?$	Enter degree of the polynomial
3[R/S]	$a0,a1,a2=?$	Enter a_0=12121, a_1=-205, a_2=-37
12121[ENTER]-205[ENTER]-37		
[R/S]	$X1=31,00$	Root 1 (in Y register)
[R/S]	$X2=-17,00$	Root 2 (in Y register)
[R/S]	$X3=23,00$	Root 3 (in Y register)

Find the (complex) roots of the following polynomial: $f(x) = x^3 - 10x^2 + x + 150$

KEYSTROKES	DISPLAY	COMMENTS
[R/S]	$0,00000$	Run POLY234 for 3rd degree

KEYSTROKES	DISPLAY	COMMENTS
[R/S]	$DEGREE=?$	Enter degree of the polynomial
3 [R/S]	$a\ 0, a\ 1, a\ 2=?$	Enter a_0=150, a_1=1, a_2=-10
150 [ENTER] 1 [ENTER] -10		
[R/S]	$X\ 1=6.66-0.92\ I$	Root 1 (in X and Y register)
[R/S]	$X\ 2=-3.32$	Root 2 (in Y register)
[R/S]	$X\ 3=6.66+0.92\ I$	Root 3 (in X and Y register)

Examples: 4th degree polynomials

Find the (real) roots of the following polynomial: $f(x) = x^4 - 11x^3 + x^2 + 279x - 630$

KEYSTROKES	DISPLAY	COMMENTS
		Run POLY234 for 4th degree
[XEQ] [ALPHA] POLY234 [ALPHA]	$DEGREE=?$	Enter degree of the polynomial
4 [R/S]	$a\ 0, a\ 1, a\ 2, a\ 3=?$	Enter a_0=-630, a_1=279, a_2=1, a_3=-11
-630 [ENTER] 279 [ENTER] 1 [ENTER] -11		
[R/S]	$X\ 1=3.00$	Root 1 (in Y register)
[R/S]	$X\ 2=-5.00$	Root 2 (in Y register)
[R/S]	$X\ 3=7.00$	Root 3 (in Y register)
[R/S]	$X\ 4=6.00$	Root 4 (in Y register)

Find the (complex) roots of the following polynomial: $f(x) = x^4 + 10x^3 - 26x^2 + 251x + 540$

KEYSTROKES	DISPLAY	COMMENTS
[R/S]	0.00000	Run POLY234 for 4th degree
[R/S]	$DEGREE=?$	Enter degree of the polynomial
4 [R/S]	$a\ 0, a\ 1, a\ 2, a\ 3=?$	Enter a_0=540, a_1=251, a_2=-26, a_3=10
540 [ENTER] 251 [ENTER] -26 [ENTER] 10		
[R/S]	$X\ 1=1.69$	Root 1 (in Y register)
[R/S]	$X\ 2=13.18$	Root 2 (in Y register)
[R/S]	$X\ 3=2.44+4.27\ I$	Root 3 (in X and Y register)
[R/S]	$X\ 4=2.44-4.27\ I$	Root 4 (in X and Y register)
[R/S]	$-4.2718\ 7$	Complex root 4 part in X register
[X<>Y]	2.43744	Real root 4 part in Y register
[R/S]	$DEGREE=?$	Run POLY234 for another polynomial

Program Listing

The listing of program POLY234 is given on the next page. The FIX 7 step in line 362 can sometimes lead to erroneous complex roots due to numerical limitations of the calculator's mantissa. You may have to change it to FIX 6 when working with small and large coefficient values.

```
01▪LBL "POLY234"      53 *              105 15              157 RCL 07
02 CF 02              54 +              106 STO 06          158 +
03 CF 03              55 2              107 RCL 12          159 RCL 08
04 CF 04              56 /              108 ABS             160 CHS
05 "DEGREE=?"         57 STO 12         109▪LBL 14          161 GTO 09
06 PROMPT             58 X=0?           110 2               162▪LBL 02
07 "a0,a1"            59 SF 03          111 RCL 01          163 CLST
08 GTO IND X          60 X<0?           112 ST- IND 06      164 STO 10
09▪LBL 03             61 SF 02          113 RDN             165 >"=?"
10▪LBL 04             62 RCL 05         114 RCL IND 06      166 PROMPT
11 >",a2"             63 RCL 07         115 *               167▪LBL 09
12 GTO IND X          64 3             116 LASTX           168 CF 01
13▪LBL 04             65 *              117 X^2             169 2
14 >",a3=?"           66 -              118 RCL 13          170 /
15 PROMPT             67 STO 11         119 -               171 CHS
16 STO 14             68 4              120 X<0?            172 STO 00
17 X^2                69 ST/ 14         121 GTO 12          173 X^2
18 X<>Y               70 FC? 03         122 SQRT            174 X<>Y
19 STO 05             71 GTO 11         123 STO 07          175 -
20 *                  72 RCL 13         124 X<>Y            176 X<0?
21 16                 73 X<>Y           125 RCL 11          177 SF 01
22 /                  74 X<> Z          126 -               178 ABS
23 X<>Y               75 XEQ 09         127 X<0?            179 SQRT
24 STO 06             76 CF 03          128 GTO 12          180 STO 01
25 RCL 14             77 SF 04          129 SQRT            181 FC? 04
26 *                  78 CHS            130 FS?C 02         182 GTO 02
27 4                  79 .              131 CHS             183 RCL 14
28 /                  80 STO 10         132 STO 08          184 ST- 00
29 -                  81 XEQ 09         133 *               185 RDN
30 +                  82 CLST           134 ABS             186▪LBL 02
31 RCL 14             83 RCL 07         135 X<>Y            187 RCL 00
32 X^2                84 CHS            136 -               188 FC? 01
33 RCL X              85 .              137 ABS             189 +
34 8                  86 GTO 09         138 .1              190 FS? 03
35 /                  87▪LBL 11         139 X>Y?            191 STO 07
36 STO 07             88 RDN            140 GTO 13          192 FC? 03
37 *                  89 2              141 LASTX           193 XEQ 08
38 3                  90 /              142 STO Z           194 CLST
39 *                  91 STO Y          143▪LBL 12          195 RCL 01
40 32                 92 RCL 13         144 RCL Z           196 CHS
41 /                  93 *              145 1               197 RCL 00
42 -                  94 RCL 12         146 ST+ 06          198 FC?C 01
43 STO 13             95 X^2            147 RDN             199 +
44 RCL 06             96 2              148 GTO 14          200 FS? 03
45 RCL 14             97 /              149▪LBL 13          201 RTN
46 RCL 05             98 -              150 .               202 GTO 08
47 *                  99 RCL 13         151 RCL IND 06      203▪LBL 03
48 2                  100 CHS           152 RCL 07          204 >"=?"
49 /                  101 RCL Z         153 -               205 PROMPT
50 -                  102 CHS           154 RCL 08          206▪LBL 10
51 RCL 07             103 SF 04         155 XEQ 09          207 CF 00
52 RCL 14             104 XEQ 10        156 RCL IND 06      208 STO 00
```

209 3	255 Y^X	301 STO 04	347▪LBL 07
210 /	256 X<>Y	302 X<>Y	348 X<>Y
211 STO 01	257 RCL 03	303 STO 05	349 RDN
212 RCL 00	258 *	304 RCL 09	350 +
213 *	259 X<>Y	305 RCL 08	351 RDN
214 X<>Y	260 P-R	306 RCL 07	352 +
215 ST- Y	261 STO 04	307 RCL 06	353 R^
216 RCL 01	262 X<>Y	308 XEQ 06	354 RTN
217 *	263 STO 05	309 STO 06	355▪LBL 05
218 CHS	264 X<>Y	310 X<>Y	356 RCL 01
219 R^	265 RCL 02	311 STO 07	357 -
220 +	266 -3	312 X<>Y	358▪LBL 08
221 2	267 /	313 RCL 09	359 ISG 10
222 /	268 .	314 RCL 08	360 DEG
223 RCL 01	269 STO 10	315 XEQ 06	361 CF 29
224 3	270 X<>Y	316 RCL 05	362 FIX 07
225 Y^X	271 R^	317 RCL 04	363 RND
226 +	272 R^	318 XEQ 07	364 X<>Y
227 X<>Y	273 R-P	319 STO 16	365 RND
228 CHS	274 1/X	320 FC? 04	366 FIX 00
229 STO 02	275 X<>Y	321 XEQ 05	367 "X"
230 3	276 CHS	322 RCL 09	368 ARCL 10
231 /	277 X<>Y	323 RCL 08	369 >"="
232 3	278 P-R	324 RCL 05	370 FIX 02
233 Y^X	279 XEQ 06	325 RCL 04	371 SF 29
234 RCL Y	280 STO 06	326 XEQ 06	372 X<>Y
235 X^2	281 X<>Y	327 RCL 07	373 X=0?
236 +	282 STO 07	328 RCL 06	374 SF 00
237 X<0?	283 X<>Y	329 XEQ 07	375 X<>Y
238 SF 00	284 RCL 05	330 STO 17	376 FS?C 00
239 ABS	285 RCL 04	331 FS? 04	377 GTO 08
240 SQRT	286 XEQ 07	332 RTN	378 ARCL Y
241 X<>Y	287 STO 15	333 GTO 05	379 X=0?
242 CHS	288 FC? 04	334▪LBL 06	380 GTO 09
243 FS?C 00	289 XEQ 05	335 R-P	381 X>0?
244 GTO 00	290 3	336 R^	382 >"+"
245 +	291 SQRT	337 R^	383▪LBL 08
246 .	292 .5	338 R-P	384 ARCL X
247 X<>Y	293 *	339 X<>Y	385 X#0?
248▪LBL 00	294 STO 09	340 RDN	386 >"I"
249 3	295 LASTX	341 *	387▪LBL 09
250 1/X	296 CHS	342 RDN	388 FIX 05
251 STO 03	297 STO 08	343 +	389 PROMPT
252 RDN	298 RCL 05	344 R^	390 END
253 R-P	299 RCL 04	345 P-R	
254 RCL 03	300 XEQ 06	346 RTN	(542 bytes)

Registers, Labels and Flags

REGISTERS	COMMENTS
R00-R14	Work registers, general use

LABELS	COMMENTS
LBL00	Negative square root
LBL01	Not used

		LBL02	Handle 2nd degrees (reused)
		LBL03	Handle 3rd degrees (reused)
		LBL04	Handle 4th degrees (reused)
		LBL05	For 3rd degrees
		LBL06	For 4th degrees
		LBL07	Common general handlings
		LBL08	Display of root values
		LBL09-14	Process specific conditions

FLAGS	COMMENTS
00	Indicate negative value before square root determination
01	Indicate complex root 2nd degree
02	Indicate complex root 4th degree
03	Indicate complex root 3rd degree
04	Indicate complex root 4th degree

Downloads

The RAW/TXT format of the program is available via the website: POLY234 (in zip file).